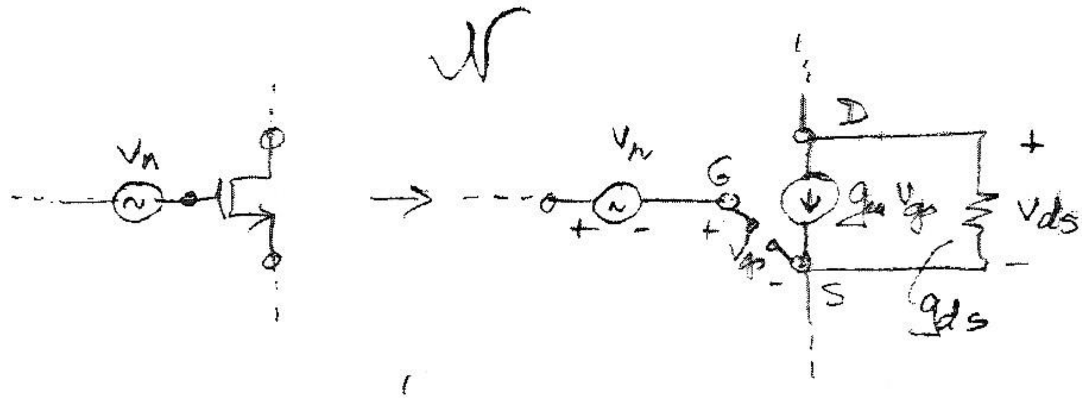
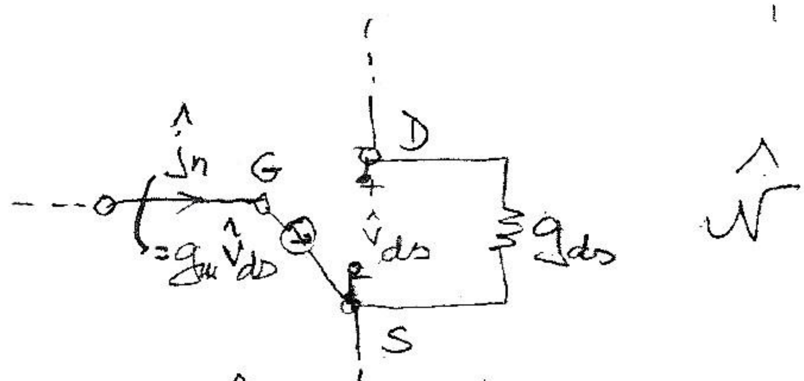
Noise analyisis of MOSFET IC:




$$\frac{\partial V_{out}}{\partial V_n} = -\hat{j}_n = -g_m \hat{v}_{ds}$$

$$\overline{v}_{out,n}^2 \cong \sum_k \left(\frac{\partial v_{out}}{\partial v_{n_k}}\right)^2 \overline{v}_{nk}^2 = \sum (g_{m_k} \hat{v}_{ds_k})^2 \overline{v}_{nk}^2$$

Frequency resp. needed.

# Sensitivity Calculations (DC) for Nonlinear Networks

Consider a nonlinear conductance:


$i \leftrightarrow j$

where $i_B = i_B(V_B, P_1, P_2, \ldots)$.

Here the $P_i$ may be time ($t$), temperature (T), α, $I_S$, etc.

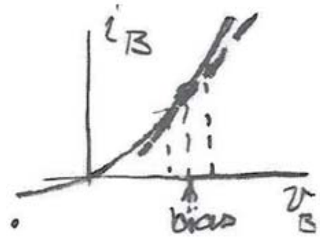
Then $\Delta i_B \cong \frac{\partial i_B}{\partial V_B}\Delta V_B + \sum_k \frac{\partial i_B}{\partial P_k}\Delta P_k$

The contribution to Δ is [diff. Tellegen theorem $\Delta = \sum_k(\hat{V}_k\,\Delta i_k - \hat{\imath}_k \Delta V_k) = 0$]

$$\hat{V}_B \Delta i_B - \hat{\imath}_B \Delta V_B = \left[\underbrace{\hat{V}_B\left(\frac{\partial i_B}{\partial V_B}\right) - \hat{\imath}_B}_{0}\right]\Delta V_B + \hat{V}_B \sum_k \frac{\partial i_B}{\partial P_k}\Delta P_k \rightarrow \frac{\partial V_{out}}{\partial P_k} = \hat{v}_B \frac{\partial i_B}{\partial P_k}$$

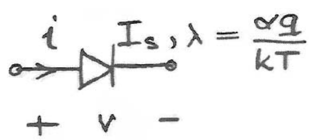
The first term is eliminated if in $\hat{N}$

| | |
|---|---|
| $\hat{\imath}_B = \left(\frac{\partial i_B}{\partial V_B}\right) \hat{V}_B$ ($\hat{g}$) *bias point* | $R = {}^1/_{\hat{g}}$  Incremental linear resistor |




This represents a linear time-variable conductance $\hat{g}$ in $\hat{N}$. (Time-variable, since $\partial i_B/\partial V_B$ varies in $N$ in time, even if the $i_B$-$V_B$ characteristic is time-invariant, except of course for DC circuits.) Or for small signals!

Example:

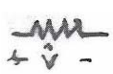



$n$ or α: production variable, 1~2.

sat. current, λ

$$i = I_S\left(e^{\left(\frac{\alpha g}{kT}\right)v} - 1\right)$$

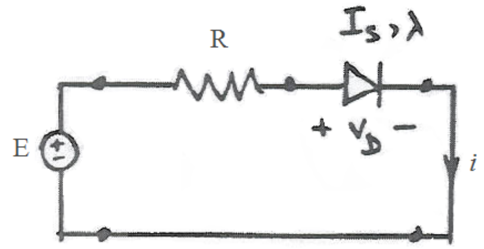
Hence, $\hat{g} = \frac{\partial i}{\partial v} = \lambda(i + I_S)$




$$P_1 = I_S, P_2 = T, P_3 = \alpha;\ \ \hat{g} = \frac{\partial i}{\partial v} = \frac{\alpha q I_S}{kT} e^{\frac{\alpha q v}{kT}} = \frac{\alpha q (i + I_S)}{kT} = \lambda(i + I_S)$$

The contribution to Δ is $\hat{v}\sum_k \frac{\partial i}{\partial P_k}\Delta P_k = \hat{v}\left[\frac{i}{I_S}\Delta I_S - \frac{\alpha q v(i+I_S)}{kT^2}\Delta T + \frac{qv(i+I_S)}{kT}\Delta\alpha\right] + \cdots = \Delta v_0$

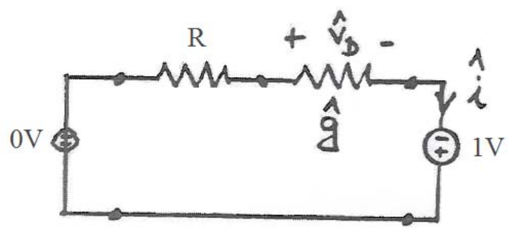
Example: Find the temperature sensitivity of ***i*** in the circuit:




Temp. sensor R is negligibly temp. sensitive

$$\Delta i = \frac{\partial i}{\partial T}\Delta T$$

Solution: now $\hat{N}$ is




This gives $\frac{[\hat{i}_I^T]}{[\hat{j}_i^T]}\frac{[\Delta V_I]}{[\Delta v_i]} - \frac{[\hat{V}_V^T]}{[\hat{e}^T]}\frac{[\Delta i_V]}{[\Delta j_e]} = \Delta i$ on the LHS of the diff. Tellegen's Theorem.

If only T varies, the RHS is $\frac{\partial i}{\partial T}\Delta T = -\hat{V}_D\,\overbrace{\lambda\frac{V_D(i+I_S)}{T}}^{\partial i/\partial T}\Delta T$

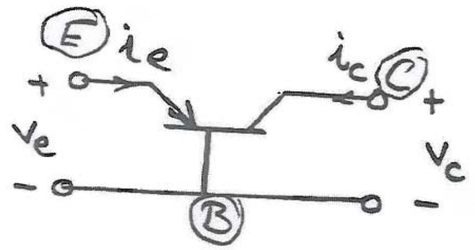
where $\hat{V}_D = +\frac{1/\hat{g}}{R+1/\hat{g}} = \frac{1}{R\hat{g}+1} = \frac{1}{R\underbrace{\lambda(i+I_S)}+1}$, so that

$$\Delta i = \overbrace{-\frac{V_D\lambda(i+I_S)}{R\lambda(i+I_S)+1}\frac{\Delta T}{T}}^{\frac{\partial i}{\partial T}} = S_i^T\Delta T$$

Homework: find $\partial i/\partial I_S$ and $\partial i/\partial R$ .

Let a nonlinear internal twoport (e.g., a transistor) inside *N* be described by

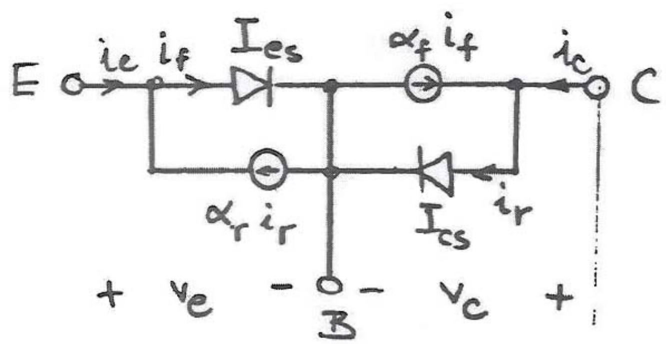
$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \boldsymbol{i} = \boldsymbol{h}(\boldsymbol{V}, P_1, P_2, \ldots)$ , where $\boldsymbol{h}$ is a non linear vector function of $\boldsymbol{V}$ and $P_i$. For example,

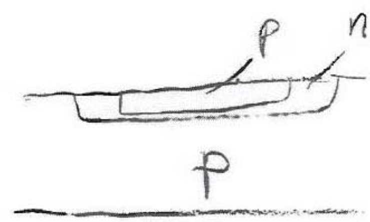



Nonlinear DC transistor model.

Using the Ebers-Moll equations (for pnp transistor)← $\begin{bmatrix} i_e \\ i_c \end{bmatrix} = \begin{bmatrix} I_{es}(e^{\lambda V_e} - 1) - \alpha_r I_{cs}(e^{\lambda V_c} - 1) \\ -\alpha I_{es}(e^{\lambda V_e} - 1) + I_{cs}(e^{\lambda V_c} - 1) \end{bmatrix}$

corresponding to the model:




Here $\boldsymbol{i} \triangleq \begin{bmatrix} i_e \\ i_c \end{bmatrix}$ ,$\boldsymbol{V} \triangleq \begin{bmatrix} V_e \\ V_c \end{bmatrix}$, $P_1 = \lambda$, $P_2 = I_{es}$, $P_3 = \alpha_f$ , etc., whichever can vary.

$\boldsymbol{i} \triangleq \begin{bmatrix} i_e \\ i_c \end{bmatrix}$ As in the scalar case, $\Delta \boldsymbol{i} = \sum_{k=1}^{2} \frac{\partial i}{\partial v_k} \Delta v_k + \sum_i \frac{\partial i}{\partial p_i} \Delta p_i$

Defining the Jacobian matrix

$$[J_v] \triangleq \frac{\partial \boldsymbol{i}}{\partial \boldsymbol{v}} \triangleq \begin{bmatrix} \frac{\partial i_1}{\partial v_1} & \frac{\partial i_1}{\partial v_2} & \cdots \\ \frac{\partial i_2}{\partial v_1} & \ddots & \vdots \\ \vdots & \cdots & \frac{\partial i_k}{\partial v_l} \end{bmatrix} \quad \text{2 x 2 for transistor}$$

k

l

$$\begin{bmatrix} \partial i_1/\partial v_1 & \partial i_1/\partial v_2 \\ \partial i_2/\partial v_1 & \partial i_2/\partial v_2 \end{bmatrix} \begin{bmatrix} \Delta v_1 \\ \Delta v_2 \end{bmatrix}$$

$$\Delta \boldsymbol{i} = [J_v]\Delta \boldsymbol{v} + \sum_i \frac{\partial \boldsymbol{i}}{\partial p_i} \Delta p_i$$

Hence, in $\hat{\boldsymbol{v}}_B^T \Delta \boldsymbol{i}_B - \hat{\boldsymbol{i}}_B^T \Delta \boldsymbol{v}_B$ , we get $\hat{\boldsymbol{v}}^T \left( [J_v]\Delta \boldsymbol{v} + \sum_i \frac{\partial i}{\partial p_i} \Delta p_i \right) - \hat{\boldsymbol{i}}^T \Delta \boldsymbol{v}$

The factor of $\Delta \boldsymbol{v}$ (which is unknown and unwanted) is

$$\hat{\boldsymbol{v}}^T [J_v] - \hat{\boldsymbol{i}}^T \Rightarrow 0 \;\; \text{for } \hat{\boldsymbol{i}} = [J_v^T]\hat{\boldsymbol{v}}.$$

$$\begin{bmatrix} \hat{i}_1 \\ \hat{i}_2 \end{bmatrix} = [J_v^T] \begin{bmatrix} \hat{v}_1 \\ \hat{v}_2 \end{bmatrix}$$

Hence, the nonlinear twoport in $N$ becomes in $\hat{N}$ a linear (time-varying) twoport given by a

$$[\hat{Y}_B] = [J_v^T]$$

$$\begin{bmatrix} \hat{i}_1 \\ \hat{i}_2 \end{bmatrix} = [J_v^T] \begin{bmatrix} \hat{v}_1 \\ \hat{v}_2 \end{bmatrix} = \begin{bmatrix} \partial i_1/\partial v_1 & \partial i_2/\partial v_1 \\ \partial i_1/\partial v_2 & \partial i_2/\partial v_2 \end{bmatrix} \begin{bmatrix} \hat{v}_1 \\ \hat{v}_2 \end{bmatrix}$$

This choice for $\hat{N}$ leaves $\hat{\boldsymbol{v}}^T \sum_i \frac{\partial \boldsymbol{i}}{\partial p_i} \Delta p_i$ as contribution of the nonlinear twoport to Δ.

For the example on p. 337, if $i_1 = i_e$, $i_2 = i_c$

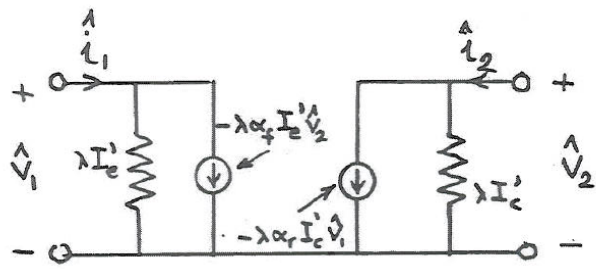
$\frac{\partial i_e}{\partial v_e}$ $\frac{\partial i_e}{\partial v_c}$

$$[J_v] = \begin{bmatrix} I_{es}\lambda e^{\lambda v_e} & -\alpha_r I_{cs}\lambda e^{\lambda v_c} \\ -\alpha_f I_{es}\lambda e^{\lambda v_e} & I_{cs}\lambda e^{\lambda v_c} \end{bmatrix}$$

So, denoting $I_{es}e^{\lambda v_e} = I_e'$, $I_{cs}e^{\lambda v_c} = I_c'$

$$[\hat{Y}_B] = \lambda \begin{bmatrix} I'_e & -\alpha_f I'_e \\ -\alpha_r I'_c & I'_c \end{bmatrix} \cdot \begin{bmatrix} \hat{\imath}_1 \\ \hat{\imath}_2 \end{bmatrix} = [\hat{Y}_B] \begin{bmatrix} \hat{v}_1 \\ \hat{v}_2 \end{bmatrix}$$

This represents the linear (time-varying) twoport:




Since $I'_e = f_1(v_e)$ & $I'_c = f_2(v_c)$, the elements in this linear twoport vary with the voltages in *N*. Hence, *N* must be analyzed before $\hat{N}$; then $\hat{N}$ is easy to analyze, since it is linear.

The contribution to Δ is, with

$$\lambda = \frac{\alpha q}{kT},\ \ p_1 = I_{e_s},\ \ p_2 = \underbrace{\alpha}_{\text{prod. paramter}},\ \ p_3 = T,\ \ p_4 = \alpha_r,\ \ p_5 = I_{c_s},\ \ p_6 = \alpha_f:$$

$$[\hat{v}_1 \quad \hat{v}_2] \begin{bmatrix} \sum_{l=1}^{6} \frac{\partial i_e}{\partial p_l} \Delta p_l \\ \sum_{l=1}^{6} \frac{\partial i_c}{\partial p_l} \Delta p_l \end{bmatrix} = \hat{v}_1 \sum_{l=1}^{6} \frac{\partial i_e}{\partial p_l} \Delta p_l + \hat{v}_2 \sum_{l=1}^{6} \frac{\partial i_c}{\partial p_l} \Delta p_l$$

Homework: calculate the terms under the summation signs.

## Sensitivity Analysis of Dynamic Linear Active Circuits in the Frequency Domain

$$v(t), j(t) \ \rightarrow V(\omega), J(\omega)$$

$$v(t) = V_v \cos(\omega t + \phi) \rightarrow V e^{j\phi},\ \ V = V_v e^{j\phi}$$